

# Spying on photons with photons: quantum interference and information

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**Abstract.** The quest to have both which-path knowledge and interference fringes in a double-slit experiment dates back to the inception of quantum mechanics (QM) and to the famous Einstein-Bohr debates. In this paper we propose and discuss an experiment able to spy on one photon's path with another photon. We modify the quantum state inside the interferometer as opposed to the traditional physical modification of the “wave-like” or “particle-like” experimental setup. We are able to show that it is the ability to harvest or not which-path information that finally limits the visibility of the interference pattern and not the “wave-like” or “particle-like” experimental setups. Remarkably, a full “particle-like” experimental setup is able to show interference fringes with 100 % visibility if the quantum state is carefully engineered.

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## 1 Introduction

Discussions about the wave-particle duality and the counter-intuitive features of quantum mechanics started almost a century ago with the famous Einstein-Bohr debates [1].

It was generally accepted that the loss of interference in a two-slit experiment is a consequence of the perturbation induced by the measuring device [2], perturbation lower bounded by the Heisenberg uncertainty principle. Bohr [3] was the first to realize that Quantum Mechanics is more subtle than this “observation caused disturbance induces the uncertainty” dictum and his Complementary Principle refined the so-called wave-particle duality to a new level. The detailed quantum mechanical treatment of Einstein's recoiling slit experiment was done by Wootters and Zurek [4]. The authors also discuss the partial which-path knowledge and the reduced contrast of the interference fringes, thus bringing a quantitative discussion to Bohr's complementarity principle. The inequality  $K^2 + V^2 \leq 1$  (where  $K$  quantifies the which-path information and  $V$  the interference fringe visibility) was successively refined and discussed by many authors [4,5,6,7,8].

A couple of decades after Bohr's proposal, Wheeler [9, 10] introduced the idea of delayed-choice experiment. In his original *Gedankenexperiment* one could decide to remove or not the second beam splitter in a Mach-Zehnder interferometer (MZI) after the light quantum left the first beam splitter. This delayed choice decides on the “wave-like” or “particle-like” phenomenon that is measured. And if this decision is space-like separated with the passage of the light quantum in the first beam splitter, no causal link could exist between these two events. It took several

decades until Jacques *et al.* [11,12] fully tested Wheeler's original delayed choice proposal. However, physicists had little doubts of its outcome and previous experiments tested equivalent schemes. Using extremely attenuated laser light, Hellmuth, Walther, Zajonc and W. Schleich [13] performed such an experiment. Their results show “no observable difference between normal and delayed-choice modes of operation”, as predicted by quantum mechanics. Two years later, using single-photon states Balduhn, Mohler and Martienssen [14] performed the same experiment and arrived at a similar conclusion. Complementarity was also considered with atoms scattering light [15,16,17] up to the quantum classical boundary [18].

A new twist in this experiment arrived with the proposal of Ionicioiu and Terno [19], where – in a truly quantum mechanical style – the second beam splitter is in a superposition of being and not being inserted. One can therefore morph between “wave” and “particle” behavior in a continuous manner. Experimental verifications followed by Kaiser *et al.* [20] and Peruzzo *et al.* [21].

Meanwhile the new idea of *Quantum Eraser* was proposed by Scully and Drühl [22]. In fact, it was possible to “erase” a which path information and – surprisingly – revive the interference fringes previously washed out by the which-path markers [23,24,25]. The original experiment with emitting atoms and micromaser cavities was deemed too difficult to implement, therefore focus was set on equivalent optical implementations [26,27]. Even a do-it-yourself quantum eraser has been suggested [27]. For a review of delayed-choice and quantum eraser proposals and experiments see reference [28].

Using a bi-photon state to realize a quantum eraser seems to be first proposed by Ou [29] by using two parametric down-converters and the “phase memory” of the pumping beam [30]. Recently, in [31] three non-linear crystals were employed to show the complementarity between the visibility of interference fringes and the which-path knowledge, proving that this subject is still an active research topic.

In this paper we revisit and modify the experiment proposed in reference [32]. The main idea – spying on photons inside a MZI with photons leaking out of it – is thoroughly discussed and taken one step further. We are able to show that one does not need a quantum eraser to recover interference: if the quantum state inside the MZI is carefully engineered so that no which-path information can be inferred from the “leaked” photon, interference fringes are always recovered, even in a full “particle-like” experimental setting. Similar to previous proposals [19,20], a continuous morphing between wave-like and particle-like behavior is also possible. This time, however, we have a *fixed* experimental setup and we morph between particle-like and wave-like behavior by modifying the input state vector.

The paper is organized as follows. The classic subject of quantum interference and path information with or without delayed choice in a MZI is given in Section 2. The idea of spying on photons with photons is introduced and discussed in Section 3. The input state is modified in Section 4 so that the which-path information obtainable from the inner MZI can be varied from zero to maximal. Finally conclusions are drawn in Section 5.

## 2 Interference and which-path information with a Mach-Zehnder interferometer

A Mach-Zehnder interferometer is depicted in Fig. 1. It is composed of the beam splitters  $BS_1$  and  $BS_2$ , together with the mirrors  $M_1$  and  $M_2$ . The beam splitters are assumed identical and are characterized by the transmission (reflection), coefficients  $T$  ( $R$ ). Unitarity constrains imply  $|T|^2 + |R|^2 = 1$  and  $RT^* + TR^* = 0$  [33].

The delay  $\varphi$  models the path length difference of the interferometer. Detectors  $D_4$  and  $D_5$  are placed at the two outputs of beam splitter  $BS_2$ . Throughout this paper, we shall assume monochromatic light and ideal photo-detectors.

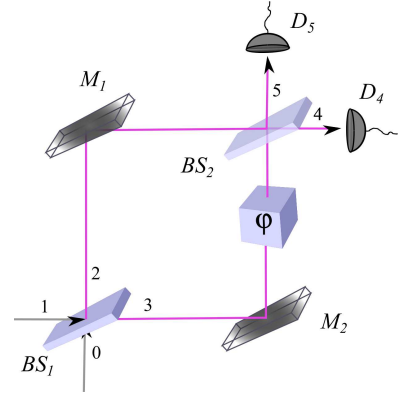
When dealing with a balanced (50/50) beam splitter, we shall use  $T = 1/\sqrt{2}$  and  $R = i/\sqrt{2}$ .

If we apply a single photon Fock state at the input 1 of our interferometer, we can write the input state as

$$|\psi_{in}\rangle = |1_1 0_0\rangle = \hat{a}_1^\dagger |0\rangle \quad (1)$$

where  $\hat{a}_k^\dagger$  denotes the creation operator at the port  $k$ . After the first beam splitter, the state vector can be written as [33]

$$|\psi_{23}\rangle = R|1_2 0_3\rangle + T|0_2 1_3\rangle \quad (2)$$



**Fig. 1.** The Mach-Zehnder interferometer. The phase shift  $\varphi$  models a voluntarily introduced path length difference. Wheeler’s delayed choice can be added as suggested by the semi-transparent beam splitter  $BS_2$ .

This is an entangled state [34]. Our photon is no more in a definitive path, it is in a coherent superposition of being in both the upper and lower paths (with probabilities  $|R|^2$  and, respectively,  $|T|^2$ ). Using the field operator transformation (see e. g. [33], [35])

$$\hat{a}_1^\dagger = TR(1 + e^{i\varphi})\hat{a}_4^\dagger + (T^2 e^{i\varphi} + R^2)\hat{a}_5^\dagger \quad (3)$$

one can find right away the output state vector

$$|\psi_{out}\rangle = TR(1 + e^{i\varphi})|1_4 0_5\rangle + (T^2 e^{i\varphi} + R^2)|0_4 1_5\rangle \quad (4)$$

and if both beam splitters are balanced (50/50), we get

$$|\psi_{out}\rangle = \cos(\varphi/2)|1_4 0_5\rangle + \sin(\varphi/2)|0_4 1_5\rangle \quad (5)$$

where we neglected a common phase factor. The probability of photo-detection at, say, detector  $D_4$  is simply  $P_4 = |\langle 1_4 0_5 | \psi_{out} \rangle|^2$  yielding

$$P_4 = \cos^2(\varphi/2) = \frac{1 + \cos(\varphi)}{2} \quad (6)$$

We recognize here the well-known interference fringes, having 100 % visibility. We had no which-path knowledge whatsoever, therefore following Feynman’s rules the (complex) amplitudes corresponding to the two paths are added up, yielding this interference phenomenon.

Starting from equations (1)–(4) we can define the which-path information as

$$K = ||T|^2 - |R|^2| \quad (7)$$

and the interference fringe visibility

$$V = 2|T||R|. \quad (8)$$

and a short computation yields immediately  $K^2 + V^2 = 1$ .

As a final remark, equation (2) states that our single photon is in a coherent superposition of being in both arms at the same time. This is not just a metaphor and one could easily convince oneself by changing the input

state so that the state vector after the beam splitter  $BS_1$  becomes e. g.

$$|\psi_{23}\rangle = |1_2 0_3\rangle \quad (9)$$

This time our light quantum is definitely in the upper arm of the interferometer, however it is a simple exercise to show that with this modification no interference can be expected from our MZI.

Delayed choice can be added in the spirit of Wheeler [9], where the decision to insert or not the second beam splitter is delayed so that there is a space-like separation between the passage of our light quantum through  $BS_1$  and the insertion/removal of  $BS_2$ . Experimental results are unambiguous [11, 13, 14]: there is no difference between “normal” and “delayed choice” modes, therefore there can be no decision taken beforehand by our light quantum. One can even continuously morph between “wave” and “particle” behavior [19], results [20, 21] impossible to explain with classical or hidden variable theories.

One could still speculate if there is a way of spying on the photons without disturbing them. The answer is positive and in the following we modify the experiment from Fig. 1 and show that this action – spying on the photons – is realistic and realizable.

### 3 Spying on photons with photons

The experimental setup from Fig. 1 is modified in the following way: the mirrors  $M_1$  and  $M_2$  are replaced by the beam splitters denoted  $BS_3$  and  $BS_4$ . Therefore, photons may “leak” through these beam splitters (by transmission), bringing therefore with them (under certain circumstances) the which-path information. To make the experiment more versatile, these “leaked” photons can be brought together into a new beam splitter, denoted  $BS_5$ . Instead of inserting or not with a delayed choice this beam splitter, we assume that its (transmission and reflection) parameters can be modified at will.

We end up with the experimental setup depicted in Fig. 2. This time, at the two inputs of the beam splitter  $BS_1$  we apply the state

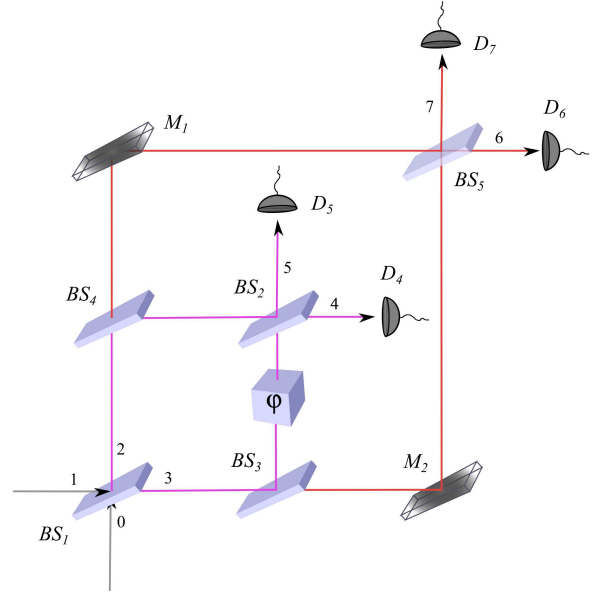
$$|\psi_{in}\rangle = |1_0 1_1\rangle = \hat{a}_0^\dagger \hat{a}_1^\dagger |0\rangle \quad (10)$$

i.e. two simultaneously impinging single-photon Fock states. If the beam splitter  $BS_1$  is balanced, the wave vector at its output is

$$|\psi_{23}\rangle = \frac{i}{\sqrt{2}}|2_2 0_3\rangle + \frac{i}{\sqrt{2}}|0_2 2_3\rangle \quad (11)$$

This fact is central to the whole experiment. Equation (11) implies that if, for example, one photon is detected in the lower arm, then *with certainty* the other one is in the same arm, too. Therefore, one photon can be forced to interfere with itself while the other one can be used as which-path marker.

Beam splitters  $BS_1$  through  $BS_4$  are assumed to be balanced. The beam splitter  $BS_5$ , is characterized by the transmission (reflection) coefficients  $T_w$  ( $R_w$ ). Moreover,



**Fig. 2.** Replacing the two mirrors from Fig. 1 with beam splitters so that photons can “leak” outside, brings us to the new experimental setup. The outer MZI has the beam splitter  $BS_5$  characterized by the transmission (reflection) coefficient  $T_w$  ( $R_w$ ). These parameters can be chosen in a delayed-choice manner.

we assume that these coefficients can be chosen at any moment (delayed choice). For example, if  $T_w = 1$  ( $T_w = 0$ ) we have a configuration where the path from the mirror  $M_1$  leads directly to the detector  $D_6$  ( $D_7$ ). We have therefore maximum which-path knowledge. If  $T_w = 1/\sqrt{2}$ , a photon detected at  $D_6$  (or  $D_7$ ) could have come with equal likelihood from any path, therefore we have zero which-path information.

Starting from the input state (10) and applying the field operator transformations (see details in Appendix A) takes us to the output state vector

$$|\psi_{out}\rangle = |\psi_{inner}\rangle + |\psi_{cross}\rangle + |\psi_{outer}\rangle \quad (12)$$

where  $|\psi_{inner}\rangle$  is the part of the wavevector corresponding to both photons being inside the inner MZI,  $|\psi_{cross}\rangle$  corresponds to the part of the wavevector where one photon is inside the inner MZI and the other one inside the outer one etc. The interesting part is found in  $|\psi_{cross}\rangle$  and we have

$$|\psi_{cross}\rangle = -\frac{T_w + iR_w e^{i\varphi}}{2\sqrt{2}}|1_4 1_6\rangle - \frac{iT_w e^{i\varphi} + R_w}{2\sqrt{2}}|1_4 1_7\rangle - \frac{iT_w + R_w e^{i\varphi}}{2\sqrt{2}}|1_5 1_6\rangle - \frac{T_w e^{i\varphi} + iR_w}{2\sqrt{2}}|1_5 1_7\rangle \quad (13)$$

One could focus on one of the coincidence detections e.g. at detectors  $D_4$  and  $D_6$ ,

$$P_{4-6} = |\langle 1_4 1_6 | \psi_{out} \rangle|^2 = |\langle 1_4 1_6 | \psi_{cross} \rangle|^2. \quad (14)$$

We shall introduce a parameter  $\varepsilon \in [0, 1]$  characterizing the beam splitter  $BS_5$  and we make the choice  $T_w = \varepsilon$

and  $R_w = i\sqrt{1 - \varepsilon^2}$ . Therefore, after a short computation equation (14) becomes

$$P_{4-6} = \frac{1 - 2\varepsilon\sqrt{1 - \varepsilon^2} \cos(\varphi)}{8} \quad (15)$$

For  $T_w = 0$  (or  $T_w = 1$ ) the which-path information of the inner photon is known *with certainty* through the outer one detected at  $D_6$ . The coincidence probability now gives

$$P_{4-6} = \frac{1}{8} \quad (16)$$

As expected, all interference is gone. On the other hand maximum uncertainty on the path taken by the photon in the inner MZI is obtained when  $BS_5$  is balanced ( $\varepsilon = 1/\sqrt{2}$ ) yielding in this case

$$P_{4-6} = \frac{1 - \cos(\varphi)}{8} \quad (17)$$

i.e. interference fringes with 100% visibility. For other values of  $\varepsilon$  one has partial information about the path quantified by

$$K = ||T_w|^2 - |R_w|^2| = |2\varepsilon^2 - 1| \quad (18)$$

and interference fringes with a visibility given by

$$V = 2|T_w||R_w| = 2\varepsilon\sqrt{1 - \varepsilon^2}. \quad (19)$$

It is easy to show that we have

$$K^2 + V^2 = 1 \quad (20)$$

for all values of  $\varepsilon$  and we obtained again the extreme case of the well-known inequality quantifying the duality between which-path information and interference [4, 5, 6, 7, 8].

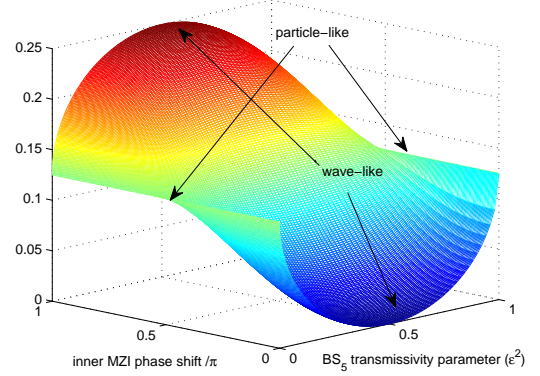
The coincidence counts probability  $P_{4-6}(\varphi, \varepsilon)$  at detectors  $D_4$  and  $D_6$  is plotted in Fig. 3. As it can be seen, we can continuously morph from “particle-like” to “wave-like” behavior by modifying the transmission parameter  $\varepsilon$  of the beam splitter  $BS_5$ .

The interesting point about this experiment is that the detection event at  $D_4$  can take place much earlier than the detection event at  $D_6$ , one can even make these event space-like separated, therefore any interpretation of pilot waves or other signals travelling both routes in the inner interferometers are simply not tenable. The decision of what aspect (wave, particle or a little of both) to watch is taken much later, long after the photon in the inner path has been detected and destroyed. The decision what to measure is in the hands of the experimenter.

It is noteworthy that ignoring the inner detectors and focusing on singles detections at, say,  $D_6$  or  $D_7$  will show no interference (see computational details in Appendix B).

#### 4 Varying the which-path information with a modified input state

The previous section ended with the conclusion that spying on the photon from the inner MZI with 100% certainty



**Fig. 3.** The coincidence probability at  $D_4$  and  $D_6$  as a function of the path length difference  $\varphi$  and the “delayed choice” beam splitter  $BS_5$  parameter  $\varepsilon^2$ . As  $\varepsilon$  goes from 0 through  $1/\sqrt{2}$  to 1 we go from maximum particle-like through maximum wave-like and back again to maximum particle-like behavior.

prevented us to observe interference effects. As stated earlier, the crucial point in our experiment was given by equation (11). Namely, we had absolute certainty that if one photon is in one arm of the interferometer, the other one will be there, too.

There is another state that is interesting in this respect, namely the input state

$$|\psi'_{in}\rangle = \frac{1}{\sqrt{2}}(|2_0 0_1\rangle + |0_0 2_1\rangle) \quad (21)$$

yielding after the beam splitter  $BS_1$  the state vector

$$|\psi'_{23}\rangle = i|1_2 1_3\rangle \quad (22)$$

This time we also have absolute certainty that if one photon is detected in one arm, the other one will be in the *opposite* arm. Using the same principle as before, we find

$$|\psi'_{out}\rangle = |\psi'_{inner}\rangle + |\psi'_{cross}\rangle + |\psi'_{outer}\rangle \quad (23)$$

where  $|\psi'_{cross}\rangle$  is given by

$$\begin{aligned} |\psi'_{cross}\rangle = & -\frac{iT_w e^{i\varphi} + R_w}{2\sqrt{2}}|1_4 1_6\rangle - \frac{T_w + iR_w e^{i\varphi}}{2\sqrt{2}}|1_4 1_7\rangle \\ & - \frac{T_w e^{i\varphi} + iR_w}{2\sqrt{2}}|1_5 1_6\rangle - \frac{iT_w + R_w e^{i\varphi}}{2\sqrt{2}}|1_5 1_7\rangle \end{aligned} \quad (24)$$

Computing again the coincidence probability

$$P_{4-6} = |\langle 1_4 1_6 | \psi'_{out} \rangle|^2 = |\langle 1_4 1_6 | \psi'_{cross} \rangle|^2 \quad (25)$$

takes us to

$$P_{4-6} = \frac{1 + 2\varepsilon\sqrt{1 - \varepsilon^2} \cos(\varphi)}{8} \quad (26)$$

The same conclusions from the previous section apply. One could therefore conclude that we have a “particle-like” setting, namely for  $T_w = 0$  or  $T_w = 1$  yielding again



the result from equation (16) and a “wave-like” setting for  $T_w = 1/\sqrt{2}$  yielding

$$P_{4-6} = \frac{1 + \cos(\varphi)}{8} \quad (27)$$

In the following, we will show that this is actually an *incomplete picture*. By simply changing the input state, we shall render useless this classification.

The state vector  $|\psi_{23}\rangle$  from equation (11) guaranteed us that the two photons are *always* in the same arm of the interferometer right after  $BS_1$  while  $|\psi'_{23}\rangle$  from equation (22) guarantees us that the two photons are *never* in the same arm. We could erase (partially or totally) this information by preparing an input state that is a coherent superposition of being both in  $|\psi_{in}\rangle$  and in  $|\psi'_{in}\rangle$ . Therefore, we define

$$|\psi''_{in}\rangle = \alpha|\psi_{in}\rangle + \sqrt{1-\alpha^2}|\psi'_{in}\rangle \quad (28)$$

and after the beam splitter  $BS_1$  we have the state vector

$$|\psi''_{23}\rangle = \alpha|\psi_{23}\rangle + \sqrt{1-\alpha^2}|\psi'_{23}\rangle \quad (29)$$

with  $\alpha \in [0, 1]$ . Using the same technique as before we can compute the output state vector  $|\psi''_{out}\rangle$ . One can obtain now the coincidence probability at the detectors  $D_4$  and  $D_6$  as

$$P_{4-6} = |\langle 1_4 1_6 | \psi''_{out} \rangle|^2 = |\langle 1_4 1_6 | \psi''_{cross} \rangle|^2 = \frac{1 - 2\alpha\sqrt{1-\alpha^2}\sin(\varphi) + 2(1-2\alpha^2)\varepsilon\sqrt{1-\varepsilon^2}\cos(\varphi)}{8} \quad (30)$$

where  $|\psi''_{cross}\rangle = \alpha|\psi_{cross}\rangle + \sqrt{1-\alpha^2}|\psi'_{cross}\rangle$ . We have now the path-related information

$$K = |2\alpha^2 - 1| |T_w|^2 - |R_w|^2 = |2\alpha^2 - 1| |2\varepsilon^2 - 1| \quad (31)$$

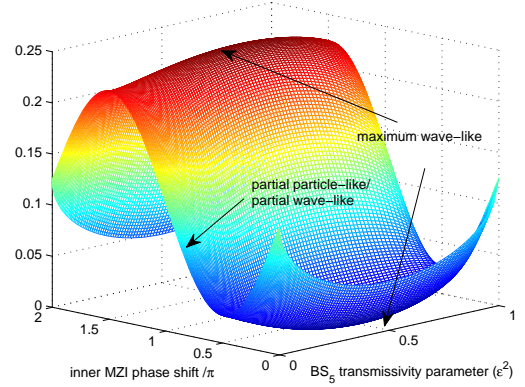
and the visibility of the interference pattern

$$V = 2\sqrt{\alpha^2(1-\alpha^2) + (2\alpha^2-1)^2 |T_w|^2 |R_w|^2} = 2\sqrt{\alpha^2(1-\alpha^2) + (2\alpha^2-1)^2 \varepsilon^2 (1-\varepsilon^2)} \quad (32)$$

One can easily check that equation (20) is again satisfied for any  $\alpha$  and  $\varepsilon$ . The least which-path information about the inner MZI photon that can be harvested by a detection with a “leaked” photon at  $D_6$  (or  $D_7$ ) from the state  $|\psi''_{in}\rangle$  is obtained for  $\alpha = 1/\sqrt{2}$ . In this case the coincidence count probability given by equation (30) becomes

$$P_{4-6} = \frac{1 - \sin(\varphi)}{8} \quad (33)$$

It is worthwhile to note that this time the coincidence probability is wave-like and does not depend on  $T_w$  ( $R_w$ ). This can be coupled by the fact that no which-path information can be extracted from equation (28) when  $\alpha = 1/\sqrt{2}$  (as proven by the fact that  $K = 0$  in this case). Therefore, it is quite remarkable that the photon from the inner interferometer is willing to show us an interference pattern with



**Fig. 4.** The coincidence counts probability  $P_{4-6}$  at  $D_4$  and  $D_6$  as a function of the path length difference  $\varphi$  and the beam splitter  $BS_5$  parameter  $\varepsilon^2$  for  $\alpha = 0.9$ . There is no more a clear “particle-like” behavior as in Fig. 3.

100% visibility, no matter if we insert or not the beam splitter  $BS_5$ . It is equally remarkable that in this case the single detection probability  $P_4$  at the detector  $D_4$  shows the same behavior.

In Fig. 4 we plot the coincidence counts probability  $P_{4-6}$  from equation (30) for  $\alpha = 0.9$ . Since less which-path information can be now extracted from the detection at  $D_6$  (we have  $K = 0.62|2\varepsilon^2 - 1|$ ) the visibility of the interference fringes is quite high. We can have a perfect “wave-like” behavior for  $\varepsilon = 1/\sqrt{2}$  (when  $K = 0$  and  $V = 1$ ), the worst case being for  $\varepsilon = 0$  (or  $\varepsilon = 1$ ) when  $V = 0.78$ .

## 5 Conclusions

In this paper reconsidered the so-called wave-particle duality from a different point of view. We extended the much-discussed simple Mach-Zehnder interferometer into two braced interferometers. The ability to convey information about the path taken by one photon immediately limits its wave-like behavior, as discussed in Section 3. Contrary to all proposed and performed experiments up-to-date, we show that it is possible to have a constant “wave-like” behavior of photon from the inner MZI no matter if the beam splitter  $BS_5$  is inserted or not by simply engineering our input state vector. We conclude that there are no pre-determined “wave-like” or “particle-like” experimental setups, it all boils down to how much information we can extract from our system.

## A Computation of the field operator transformations

In the general case, when the beam splitters are not balanced, we have the field operator transformations (see, e.

g. [32] for a similar computation):

$$\hat{a}_0^\dagger = R(T^2 + R^2 e^{i\varphi}) \hat{a}_4^\dagger + TR^2(1 + e^{i\varphi}) \hat{a}_5^\dagger + T(TT_w + RR_w) \hat{a}_6^\dagger + T(TR_w + RT_w) \hat{a}_7^\dagger \quad (34)$$

and

$$\hat{a}_1^\dagger = TR^2(1 + e^{i\varphi}) \hat{a}_4^\dagger + R(T^2 e^{i\varphi} + R^2) \hat{a}_5^\dagger + T(TR_w + RT_w) \hat{a}_6^\dagger + T(TT_w + RR_w) \hat{a}_7^\dagger \quad (35)$$

For the balanced case, the state operator transformations can be written as

$$\hat{a}_0^\dagger = \frac{e^{i\frac{\varphi}{2}} \sin(\frac{\varphi}{2})}{\sqrt{2}} \hat{a}_4^\dagger - \frac{e^{i\frac{\varphi}{2}} \cos(\frac{\varphi}{2})}{\sqrt{2}} \hat{a}_5^\dagger + \frac{T_w + iR_w}{2} \hat{a}_6^\dagger + \frac{iT_w + R_w}{2} \hat{a}_7^\dagger \quad (36)$$

and

$$\hat{a}_1^\dagger = -\frac{e^{i\frac{\varphi}{2}} \cos(\frac{\varphi}{2})}{\sqrt{2}} \hat{a}_4^\dagger - \frac{e^{i\frac{\varphi}{2}} \sin(\frac{\varphi}{2})}{\sqrt{2}} \hat{a}_5^\dagger + \frac{iT_w + R_w}{2} \hat{a}_6^\dagger + \frac{T_w + iR_w}{2} \hat{a}_7^\dagger \quad (37)$$

Taking now the input state (10) and using the field operator transformations (36) and (37) we end up with the state vector given by equation (12) where  $|\psi_{cross}\rangle$  is given by equation (13) and we have

$$|\psi_{inner}\rangle = \frac{e^{i\varphi} \sin(\varphi)}{2\sqrt{2}} (|0_4 2_5\rangle - |2_4 0_5\rangle) - \frac{e^{i\varphi} \cos(\varphi)}{2} |1_4 1_5\rangle \quad (38)$$

and

$$|\psi_{outer}\rangle = i \frac{T_w^2 + R_w^2}{2\sqrt{2}} (|2_6 0_7\rangle + |0_6 2_7\rangle) + iT_w R_w |1_6 1_7\rangle \quad (39)$$

## B Ignoring some of the detectors – the density matrix approach

If we chose to ignore everything that happens outside the inner interferometer, then we have to work in the density matrix approach. Therefore, we first construct the global density matrix  $\hat{\rho}_{out} = |\psi_{out}\rangle\langle\psi_{out}|$  and trace over the unused outputs (in this case 6 and 7), yielding the reduced density matrix

$$\hat{\rho}_{4-5} = \text{Tr}_{6,7} \{\hat{\rho}_{out}\} = \sum_{m,n \in \mathbb{N}} \langle m_6 n_7 | \hat{\rho}_{out} | m_6 n_7 \rangle \quad (40)$$

The central question is now if one could see an interference pattern while selecting only single counts (i.e. one and only one detection event at either  $D_4$  or  $D_5$ ) in the inner interferometer while completely ignoring what happens outside it. It is straightforward to check that this is

not the case. For example, the single count rate at detector  $D_4$  is

$$P_4 = \text{Tr} \left\{ \hat{a}_4^\dagger \hat{a}_4 \hat{\rho}_{4-5} \right\} = \frac{1}{2} \quad (41)$$

The reader can easily check that we get an identical answer by imposing one photo-count at  $D_5$  and none at  $D_4$  (and still ignoring the outer detectors).

## References

1. N. Bohr, *Discussions with Einstein on Epistemological Problems in Atomic Physics*, Cambridge University Press (1949)
2. R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics* (Addison-Wesley, Reading, MA, 1965), Vol. III, Chap. 1.
3. N. Bohr, *Naturwissenschaften* **16**, 245 (1928)
4. W. Wootters, W. H. Zurek, *Phys. Rev. D* **19**, 473 (1979)
5. S. Dürr, G. Rempe, *Am. J. Phys.* **68**, 1021 (2000)
6. D.M. Greenberger, A. Yasin, *Phys. Lett. A* **128**, 391 (1988)
7. B.-G. Englert, M. O. Scully, H. Walther, *Nature* **375**, 367 (1995)
8. B.-G. Englert, *Phys. Rev. Lett.* **77**, 2154 (1996)
9. J. Wheeler, *Problems in Formulation of Physics*, ed. G. t. di Francia, (North-Holland, Amsterdam, 1978)
10. J. Wheeler's "Law without law" in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, NJ, 1983)
11. V. Jacques *et al.* *Science* **315**, 966 (2007)
12. V. Jacques *et al.*, *New J. Phys.* **10**, 123009 (2008)
13. T. Hellmuth, H. Walther, A. Zajonc, W. Schleich, *Phys. Rev. A* **35**, 2532 (1987);
14. J. Balduhn, E. Mohler, W. Martienssen, *Z. Phys. B* **77**, 347 (1989).
15. M. S. Chapman *et al.* *Phys. Rev. Lett.* **75**, 3783 (1995).
16. U. Eichman *et al.* *Phys. Rev. Lett.* **70**, 2359 (1993).
17. S. Dürr, T. Nonn, G. Rempe, *Nature* **395**, 33 (1998).
18. P. Bertet *et al.* *Nature* **411**, 10 (2001)
19. R. Ionicioiu, D. R. Terno, *Phys. Rev. Lett.* **107**, 230406 (2011)
20. F. Kaiser *et al.*, *Science* **338**, 637 (2012)
21. A. Peruzzo *et al.*, *Science* **338**, 634 (2012)
22. M. O. Scully, K. Drühl, *Phys. Rev. A* **25**, 2208 (1982)
23. M. O. Scully, B.-G. Englert, H. Walther, *Nature* **351**, 111 (1991)
24. B.-G. Englert, M. O. Scully, H. Walther, *Am. J. Phys.* **67**, 4, (1999)
25. M. O. Scully, H. Walther, *Found. Phys.* **28**, 399 (1998)
26. S. P. Walborn, M. O. Terra Cunha, S. Padua, C. H. Monken, *Phys. Rev. A* **65**, 033818 (2002)
27. R. Hilmer, P. G. Kwiat, *Sci. Am.* **296**, 90 (1998)
28. X. Ma, J. Kofler, A. Zeilinger, *Rev. Mod. Phys.* **88**, 015005 (2016)
29. Z. Y. Ou, *Phys. Lett. A* **226**, 323 (1997)
30. Z. Y. Ou *et al.*, *Phys. Rev. A* **41**, 566 (1990)
31. A. Heuer, R. Menzel, P. W. Milonni, *Phys. Rev. Lett.* **114**, 053601 (2015)
32. S. Ataman, *Eur. Phys. J. D* **68**, 317 (2014)
33. R. Loudon, *The Quantum Theory of Light*, (Oxford University Press, Third Edition, 2003)
34. S. J. van Enk, *Phys. Rev. A* **72**, 064306 (2005)
35. S. Ataman, *Eur. Phys. J. D* **68**, 288 (2014)